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Dark energy models in the form of wet dark fluid in *f(R,T)* gravity

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Abstract: In this paper, the Homogenous-Hypersurface space time with wet dark fluid (WDF), which is a candidate for dark energy (DE), in the framework of *f(R, T)* gravity, *R* and *T* denote the Ricci scalar and the trace of the energy–momentum tensor, respectively (Harko et al. Phys. Rev. D, 84, 024020 (2011)) has been investigated. Equation of state in the form of WDF for the DE component of the universe has been considered. It is modeled on the equation of state $p = \omega(\rho - \rho^*)$. The exact solutions to the corresponding field equations are obtained for power-law and exponential volumetric expansion. The geometrical and physical parameters for both the models are studied.
Keywords: Dark Energy , Wet Dark Fluid .

1. Introduction

It is well known that the recent observational studies [1-5] have well established the accelerated expansion of the current universe. The universe consists of 76 % dark energy and 20 % dark matter. Several modified theories of gravity have been developed and studied to view of the late time acceleration of the Universe and the existence of dark energy and dark matter. There are various modified theories namely *f(R),f(G),f(R,G)* and *f(R,T)* .Noteworthy amongst them is the

f (R) gravity theory [6, 7]. One of the interesting and prospective versions of modified gravity theories is the *f(R,T)*gravity proposed by Harko et al. [8] wherein the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar *R* and the trace of the stress energy tensor *T* . The *f(R,T)* gravity models can explain the late time cosmic accelerated expansion of the Universe. Adhav [9] has obtained Bianchi type I cosmological model in *f (R,T)* gravity. Several Authors [10-30] studied different cosmological models in *f(R,T)* theory of gravity.

Motivated by above discussions and investigations in modified theories, it has been taken up the study of Hypersurface-Homogenous perfect fluid cosmological model in *f(R,T)* gravity. The present paper is organized as follows. A brief introduction is given in Sect. 1.In Sect. 2, a concept of wet dark fluid (WDF) has been discussed. The field equations in metric version of *f(R, T)* gravity are given in sect 3. In Sect. 4, gravitational field equation in *f(R, T)* gravity is established with the aid of the Hypersurface-Homogenous metric in the presence of WDF. The general discussion on the isotropization is given in Sect. 5.Sections 6 deals with the cosmological model for the power law. In Sect. 7, the cosmological model is discussed with exponential law of the volumetric expansion. Finally, in Sect. 8, conclusions are summarized.

2. Wet Dark Fluid (WDF).

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WDF is a new candidate for DE in the script of generalized Chaplygin gas, where a physically motivated equation of state is offered with the properties relevant for a DE problem. The equation of state for a WDF is

$$\frac{p_{WDF}}{\omega}+\rho^{*}=\rho_{WDF}. \tag{1}$$

Equation (1) is good approximation for many fluids, including water. The parameter ω and ρ^{*} are taken to be positive and restricted to $0\le\omega\le1$. Note that if c_s denotes the adiabatic sound speed in WDF, then c_s^2 [31]. To find the WDF energy density, the energy conservation equation is used

$$\rho^{*}_{WDF}+3H\left(p_{WDF}+\rho_{WDF}\right)=0. \tag{2}$$

From equation of state (1) and using $3H=\frac{\dot{V}}{V}$ in (2) equation, it's obtain

$$p_{WDF}=\left(\frac{\omega}{1+\omega}\right)\rho^{*}+\frac{c}{V^{(1+\omega)}} \quad , \tag{3}$$

where c is the constant of integration and V is the volume expansion. WDF naturally includes these components, a piece that behave as a cosmological constant as well as a standard fluid with an equation of state $p=\omega\rho$. It is shown that if we take $c>0$, this fluid will not violate the strong energy condition $p+\rho\ge0$. Thus,

$$p_{WDF}+\rho_{WDF}=(1+\omega)\rho_{WDF}-\omega\rho^{*}=(1+\omega)\left(\frac{c}{V^{(1+\omega)}}\right)\ge0 \tag{4}$$

Many Relativists [32-42] studied cosmological models with WDF in General Relativity and theories of gravitations.

3. Gravitational field equations of $f(R,T)$ gravity

The $f(R,T)$ gravity is the generalization of General Relativity (GR). In this theory, the field equations are derived from a variation, Hilbert-Einstein type principle which is given as

$$S=\frac{1}{16\pi}\int\sqrt{-g}\; f(R,T)\,d^4x+\int\sqrt{-g}\;L_m d^4x, \tag{5}$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar (R) and trace of the stress energy tensor (T) of the matter T_{ij} $(T=g^{ij}T_{ij})$ and L_m is the matter Lagrangian density.

The stress energy tensor of matter is defined as

$$T_{ij}=-\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{ij}}. \tag{6}$$

Assuming that the Lagrangian density L_m of matter depends only on the metric tensor components g_{ij} and not on its derivatives, in this case

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$$T_{ij} = g_{ij}L_m - \frac{\delta(L_m)}{\delta g^{ij}}. \tag{7}$$

The $f(R,T)$ gravity field equations are obtained by varying the action S with respect to the metric tensor components g_{ij},

$$f_R(R,T)R_{ij} - \frac{1}{2} f(R,T)g_{ij} + f_R(R,T)\left(g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_j\right) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij}, \tag{8}$$

where

$$\Theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{\alpha\beta}\frac{\partial^2 L_m}{\delta g^{ij}\partial g^{\alpha\beta}}. \tag{9}$$

Here $f_R = \frac{\delta f(R,T)}{\delta R}$, $f_T = \frac{\delta f(R,T)}{\delta T}$ $\Theta_{ij} = g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{ij}}$ and ∇_i is the covariant derivative.

The contraction of equation (8) yields

$$f_R(R,T)R + 3\Box f_R(R,T) - 2f(R,T) = \left(8\pi - f_T(R,T)\right)T - f_T(R,T)\Theta \text{ with } \Theta = g^{ij}\Theta_{ij}. \tag{10}$$

Equation (10) gives a relation between Ricci scalar and the trace of energy momentum tensor.

Using matter Lagrangian L_m the stress energy tensor of the matter is given by

$$T_{ij} = (p_{WDF} + \rho_{WDF})u_i u_j - p_{WDF} g_{ij}, \tag{11}$$

where $u^i = (0,0,0,1)$ denotes the four velocity vector in co-moving coordinates which satisfies the condition $u^i u_i = 1$. ρ_{WDF} and p_{WDF} is energy density and pressure of the fluid respectively.

The variation of stress energy of perfect fluid has the following expression

$$\Theta_{ij} = -2T_{ij} - pg_{ij}. \tag{12}$$

On the physical nature of the matter field, the field equations also depend through the tensor Θ_{ij}. Several theoretical models corresponding to different matter contributions for $f(R,T)$ gravity are possible. However, Harko et al. [8] gave three classes of these models

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R) f_3(T) \end{cases}. \tag{13}$$

In this paper, it is focused to the first class $f(R,T) = R + 2f(T)$, where $f(T)$ is an arbitrary function of tress energy tensor of the form $f(T) = \mu T$ where μ is constant. For this choice the gravitational field equations of $f(R,T)$ gravity becomes

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} - 2\dot{f}(T)T_{ij} - 2\dot{f}(T)\Theta_{ij} + f(T)g_{ij}, \tag{14}$$

where the dot denotes differentiation with respect to the argument. If the matter source is a perfect fluid then the field equations (in view of Eq. (12)) becomes

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$$R_{ij}-\frac{1}{2}Rg_{ij}=8\pi T_{ij}+2\dot{f}(T)T_{ij}+[2p_{WDF}\dot{f}(T)+f(T)]g_{ij}. \tag{15}$$

4. Field equations

the Hypersurface-Homogeneous space time of the form,

$$ds^2=dt^2-A^2(t)dx^2-B^2(t)\left[dy^2+\Sigma^2(y,K)dz^2\right], \tag{16}$$

where $A(t)$ and $B(t)$ are the cosmic scale functions, $\Sigma(y,K)=\sin y, y, \sinh y$ respectively when $K=1,0,-1$.

Hajj Boutros [43] have obtained exact solution of the field equations Using the metric (16). Solutions of Einstein field equations in the presence of perfect fluid have been discussed by Stewart and Ellis [44].The exact solutions of the field equations for Hypersurface-homogeneous space time under the assumption on the anisotropy of the fluid (dark energy) are obtained for exponential and power-law volumetric expansions in a scalar-tensor theory of gravitation by Katore and Shaikh [45]. Katore and Shaikh [46] investigated a class of solutions of Einstein's field equations describing two-fluid models of the universe in Hypersurface-Homogenous space time.

The function $f(T)$ of the trace of the stress–energy tensor of the matter is choose so that

$$f(T)=\lambda T \tag{17}$$

where λ is a constant [8].

Using comoving coordinates and equations (11)–(12) and (17), the $f(R,T)$ gravity field equations, (15), for metric (16) can be written as

$$2\frac{\ddot{B}}{B}+\left(\frac{\dot{B}}{B}\right)^2+\frac{K}{B^2}=(8\pi+3\lambda)p_{WDF}-\rho_{WDF}\lambda \tag{18}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{B}}{B}=(8\pi+3\lambda)p_{WDF}-\rho_{WDF}\lambda \tag{19}$$

$$\left(\frac{\dot{B}}{B}\right)^2+2\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{K}{B^2}=-(8\pi+3\lambda)\rho_{WDF}+p_{WDF}\lambda \tag{20}$$

where a dot hereinafter denotes ordinary differentiation with respect to cosmic time "*t*" only.

5. Isotropization and the solution

The isotropy of the expansion can be parametrized after defining the directional Hubble's parameters and the average Hubble's parameter of the expansion. The directional Hubble parameters in the directions x, y, z for the Hypersurface-Homogenous metric defined in (16) may be defined as follows:

$$H_x=\frac{\dot{A}}{A}\text{ and }H_y=H_z=\frac{\dot{B}}{B} \tag{21}$$

The mean Hubble parameter, *H*, is given by

$$H=\frac{\dot{R}}{R}=\frac{1}{3}\frac{\dot{V}}{V}=\frac{1}{3}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right) \tag{22}$$

where *R* is the mean scale factor and $V=R^3=AB^2$ is the spatial volume of the universe.

The anisotropy parameter of the expansion Δ is defined as

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$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{23}$$

in the x, y, z directions, respectively. $\Delta = 0$ corresponds to isotropic expansion.

Let us introduce the dynamical scalars, such as expansion parameter (θ) and the shear (σ^2) as usual

$$\theta = 3H \tag{24}$$

$$\sigma^2 = \frac{3}{2}\Delta H^2 \quad . \tag{25}$$

Since field equations (18)–(20) are three equations having four unknowns and are highly nonlinear, an extra condition is needed to solve the system completely. Here two different volumetric expansion laws is used

$$V = at^b \tag{26}$$

and

$$V = \alpha e^{\beta t}, \tag{27}$$

where *a*, *b*, α , β are constants. In this way, all possible expansion histories, the power law expansion, (26), and the exponential expansion, (27), have been covered.

6. Model for Power law

Here two interesting cases are discussed.

Case I a) When $B = \sqrt{V}$.

Using (26), the scale factors are obtained as follows:

$$A = 1 \tag{28}$$

and

$$B = a^{\frac{1}{2}} t^{\frac{b}{2}} . \tag{29}$$

From (3) and (1) with the help of (26), the energy density (ρ_{WDF}) and pressure (p_{WDF}) of the WDF are obtained as

$$\rho_{WDF} = \left(\frac{\omega}{1+\omega}\right)\rho^* + \frac{c}{\left(at^b\right)^{1+\omega}} \tag{30}$$

and

$$p_{WDF} = \frac{c\omega}{\left(at^b\right)^{1+\omega}} - \left(\frac{\omega}{1+\omega}\right)\rho^* . \tag{31}$$

From equation (22), the mean Hubble's parameter, *H*, is given by

$$H = \frac{b}{3t}. \tag{32}$$

The mean anisotropic parameter is obtained as

$$\Delta = \frac{1}{2}. \tag{33}$$

The Scalar expansion is given by

$$\theta = \frac{b}{t}. \tag{34}$$

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The Shear Scalar

$$\sigma^2 = \frac{b^2}{12t^2}. \qquad (35)$$

At the initial epoch, the Hubble parameter and the shear scalar are infinitely large. It is observed that the volume of the universe expands indefinitely for all positive values of *b*. The spatial volume, *V*, is zero at $t=0$. Thus the universe starts evolving with zero volume at $t=0$ and expands with cosmic time.

The deceleration parameter

$$q = \frac{3}{b} - 1 \qquad (36)$$

Case I b) When $A = V$.

Using (26), the scale factors are as follows:

$$A = at^b \qquad (37)$$

and

$$B = 1. \qquad (38)$$

From (3) and (1) with the help of (26), the energy density (ρ_{WDF}) and pressure (p_{WDF}) of the WDF are obtained as

$$\rho_{WDF} = \left(\frac{\omega}{1+\omega}\right)\rho^* + \frac{c}{\left(at^b\right)^{1+\omega}} \qquad (39)$$

and

$$p_{WDF} = \frac{c\omega}{\left(at^b\right)^{1+\omega}} - \left(\frac{\omega}{1+\omega}\right)\rho^*. \qquad (40)$$

From equation (22), the mean Hubble's parameter, *H*, is given by

$$H = \frac{b}{3t}. \qquad (41)$$

The mean anisotropic parameter is obtained as

$$\Delta = 2. \qquad (42)$$

The Scalar expansion is given by

$$\theta = \frac{b}{t}. \qquad (43)$$

The Shear Scalar

$$\sigma^2 = \frac{b^2}{3t^2}. \qquad (44)$$

The deceleration parameter

$$q = \frac{3}{b} - 1 \qquad (45)$$

At the initial epoch, the Hubble parameter and the shear scalar are infinitely large. For large t, the model tends to be isotropic. For $b > 3$ the deceleration parameter is negative. The model represents an accelerated universe.

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7. Model for exponential law

Here two interesting cases are discussed

Case I a) When $B=\sqrt{V}$.

Using (27), the scale factors are as follows:

$$A=1 \tag{46}$$

and

$$B=\alpha^{\frac{1}{2}}e^{\frac{\beta t}{2}}. \tag{47}$$

From (3) and (1) with the help of (27), the energy density (ρ_{WDF}) and pressure (p_{WDF}) of the WDF are obtained as

$$\rho_{WDF}=\left(\frac{\omega}{1+\omega}\right)\rho^{*}+\frac{c}{\left(\alpha e^{\beta t}\right)^{1+\omega}} \tag{48}$$

and

$$p_{WDF}=\frac{c\omega}{\left(\alpha e^{\beta t}\right)^{1+\omega}}-\left(\frac{\omega}{1+\omega}\right)\rho^{*}. \tag{49}$$

From equation (22), the mean Hubble's parameter, H, is given by

$$H=\frac{\beta}{3}. \tag{50}$$

The mean anisotropic parameter is obtained as

$$\Delta=\frac{1}{2}. \tag{51}$$

The Scalar expansion is given by

$$\theta=\beta. \tag{52}$$

The Shear Scalar

$$\sigma^{2}=\frac{\beta^{2}}{12}. \tag{53}$$

The deceleration parameter

$$q=-1. \tag{54}$$

The model represents an accelerated universe. Therefore the model is consistent with the cosmological observations.

Case I b) When $A=V$.

Using (27), the scale factors are as follows:

$$A=\alpha e^{\beta t} \tag{55}$$

and

$$B=1. \tag{56}$$

From (3) and (1) with the help of (27), the energy density (ρ_{WDF}) and pressure (p_{WDF}) of the WDF are obtained as

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$$\rho_{WDF} = \left(\frac{\omega}{1+\omega}\right)\rho^{*} + \frac{c}{\left(\alpha e^{\beta t}\right)^{1+\omega}} \quad (57)$$

and

$$p_{WDF} = \frac{c\omega}{\left(\alpha e^{\beta t}\right)^{1+\omega}} - \left(\frac{\omega}{1+\omega}\right)\rho^{*} \quad . \quad (58)$$

From equation (22), the mean Hubble's parameter, *H*, is given by

$$H = \frac{\beta}{3}. \quad (59)$$

The mean anisotropic parameter is obtained as

$$\Delta = 2\,. \quad (60)$$

The Scalar expansion is given by

$$\theta = \beta\,. \quad (61)$$

The Shear Scalar

$$\sigma^{2} = \frac{\beta^{2}}{3}. \quad (62)$$

The deceleration parameter

$$q = -1 \quad (63)$$

The expansion scalar, (θ) is constant throughout the evolution of the universe. The ratio of shear scalar to expansion scalar is non zero i.e. the universes is anisotropic .It is obtained the deceleration parameter $q = -1$ and $dH/dt = 0$ for this model. Hence, it gives the greatest values of the Hubble parameter and the fastest rate of expansion of the universe. The model may represent the inflationary era in the early universe and the very late time of the universe.

8. Conclusion:

Evolution of Hypersurface-Homogenous cosmological models is studied in the presence of dark energy (DE) from a wet dark fluid (WDF) in *f(R,T)* theory of gravity [8]. The exact solutions of the field equations have been obtained by assuming two different volumetric expansionlaws in a way to cover all possible expansion: namely, exponential expansion and power-law expansion. The deceleration parameter for exponential model is *q*=-1 and it predicts an accelerated expansion which resembles with Sahoo et. al.[47].

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